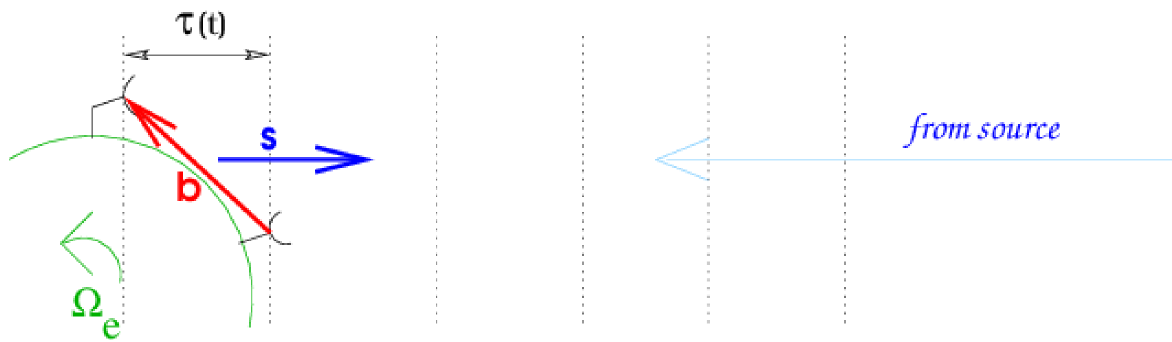
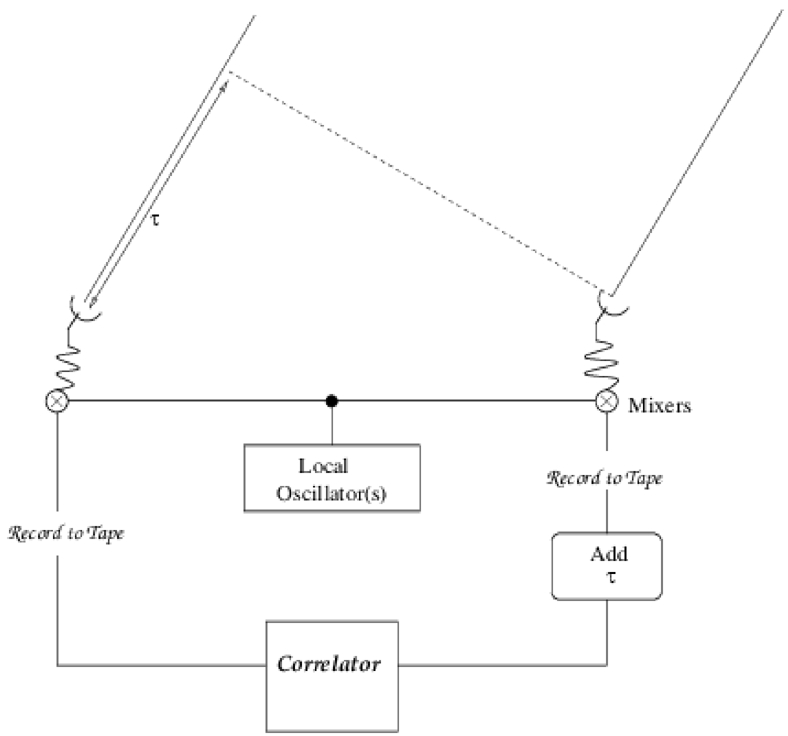
VLBI Correlation 101

Bob Campbell, JIVE

(**J**oint **I**nstitute for **V**LBI **E**RIC)

- Concepts via XF
  - Delay tracking
  - Fringe rotation
  - Bsln → Sta Based
- The shift to FX
  - SFXC at JIVE





$$\tau(t) = -\frac{1}{c}\,\mathbf{B}(t)\cdot\hat{\mathbf{s}}$$

$$\varphi(t) = \nu\tau(t)$$

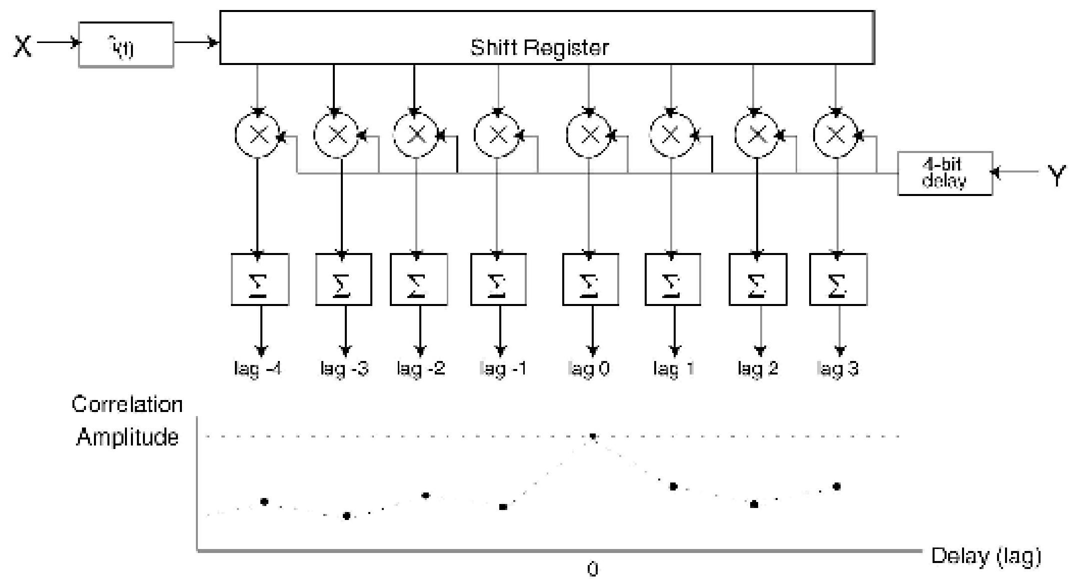
$$\tau_{\mathrm{max}} = 21.3\,\mathrm{ms} \quad (106\,\mathrm{M}\lambda)_{\mathrm{C.band}}$$

$$\dot{\tau}_{\mathrm{max}} = 1.55\,\mu\mathrm{s/s} \quad (7700\,\mathrm{cyc/s})_{\mathrm{C.band}}$$

Model Constituents (& approx. scales)

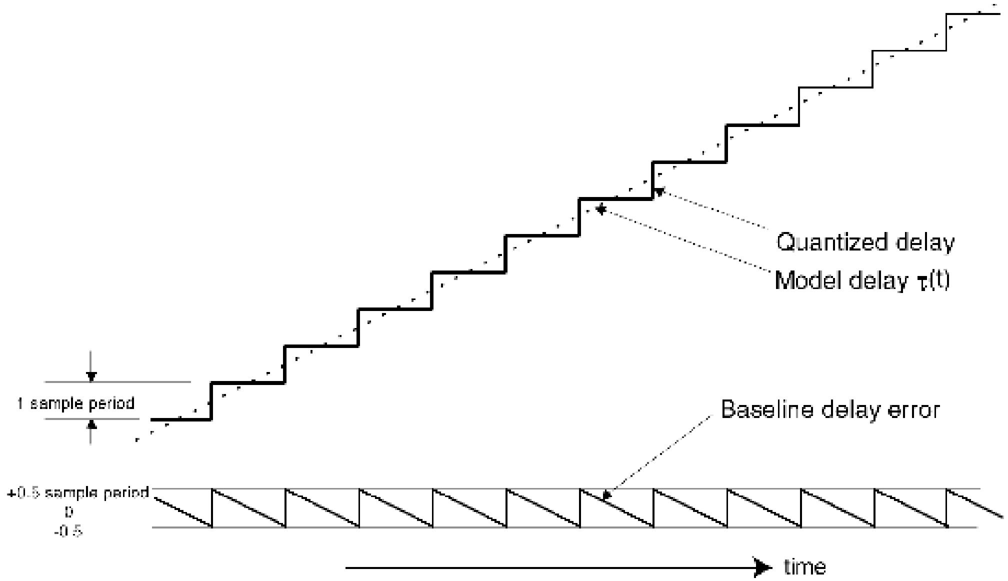
- Station / Source positions: different frames (ITRF, ICRF), motions
- Times: UTC; TAI, TT; UT1; TDB/TCB/TCG
- Orientation: Precession ($50''$/yr), Nutation ($9.6''$, 18yr), Polar Motion ($0.6''$, 1yr)
- Diurnal Spin: Oceanic friction (2ms/cy), CMB (5ms, dcds), AAM (2ms, yrs)
- Tides: Solid-earth (30cm), Pole (2cm)
- Loading: Ocean (2cm), Hydrologic (8mm), Atmospheric (2cm), PGR (mm's/yr)
- Antennas: Axis offset, Tilt, Thermal expansion
- Propagation: Troposphere (dry [7ns], wet [0.3ns]), Ionosphere
- Relativistic $\tau(t)$ calculation: Gravitational delay, Frame choice/consistency

Simplified Real (baseline) Correlator



- Shifting signals from 2 stations by a series of 1-sample steps ($\equiv$ LAGS)
  - Each lag $=$ 1 sampling interval $= \frac{1}{2BW}$ (Nyquist)
- One signal has been delayed by τ (from *a priori* model)
- This delay is constrained to be integral lags (drop/add a sample)

(baseline) Delay Tracking

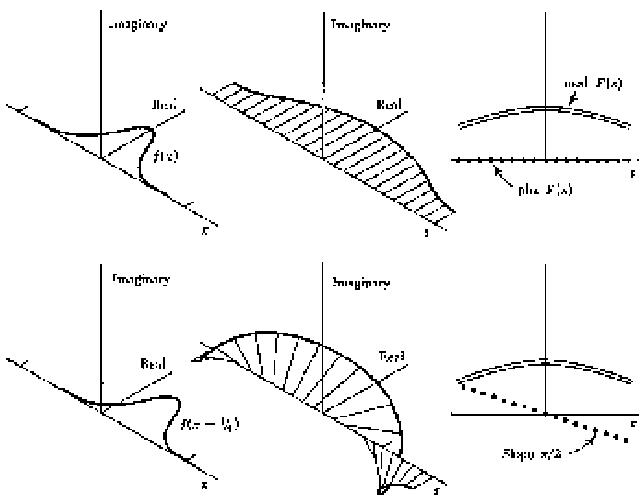


- Delay $\sim$ diurnal sinusoid
  - Amp $= B/c$; ~ 3.3 ms per 1000km of baseline
  - Delay steps $= \Delta_\tau = \frac{1}{2BW}$ (1 lag, Nyquist sampled)
- Sawtooth(y) delay error $|\delta_\tau| \leq 0.5$ lag $(\leq \frac{1}{4BW})$
- Adding/subtracting delay steps handled in bitstream

Delay error $\leftrightarrow$ φ-slope across band

⋆ Error $=$ 1 lag $\longrightarrow 180^\circ$ edge-to-edge phase slope across band

- Shift theorm: $f(t-\Delta) \rightleftharpoons e^{-i2\pi(\frac{\nu}{N_{\mathrm{lag}}})\Delta} F(\nu) \quad = \quad e^{-i\pi(\frac{\nu}{N_\nu})\Delta} F(\nu)$

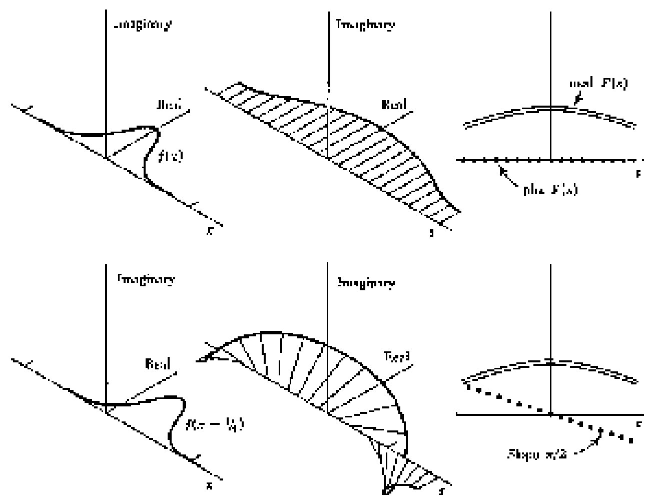


- Can also derive via:
  - Tracing expression of fringe phase (TMS §9.3.i)
  - group-delay $= \frac{1}{c}\int \mu_g\, ds = \frac{1}{c}\int \frac{d}{d\omega}(\omega\mu_p)\, ds$

Delay error ↔ φ-slope across band

⋆ Error $= 1$ lag $\longrightarrow 180°$ edge-to-edge phase slope across band

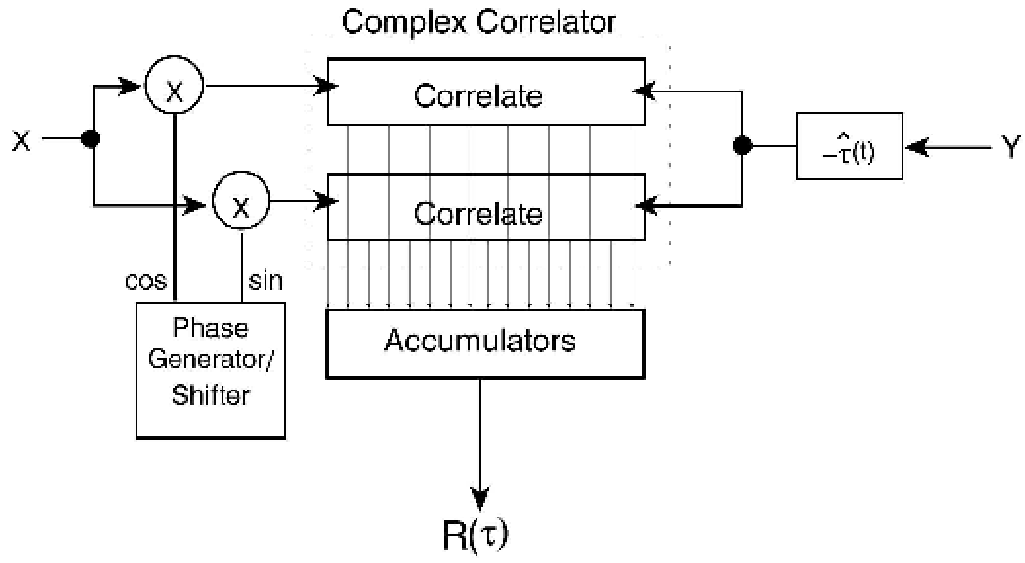
- Shift theorm: $f(t-\Delta) \rightleftharpoons e^{-i2\pi(\frac{\nu}{N_{\text{lag}}})\Delta} F(\nu) \quad = \quad e^{-i\pi(\frac{\nu}{N_{\nu}})\Delta} F(\nu)$



- Can also derive via:
  - Tracing expression of fringe phase (TMS §9.3.i)
  - group-delay $= \frac{1}{c}\int \mu_g\,ds = \frac{1}{c}\int \frac{d}{d\omega}(\omega\mu_p)\,ds$

Baseline Complex Correlator

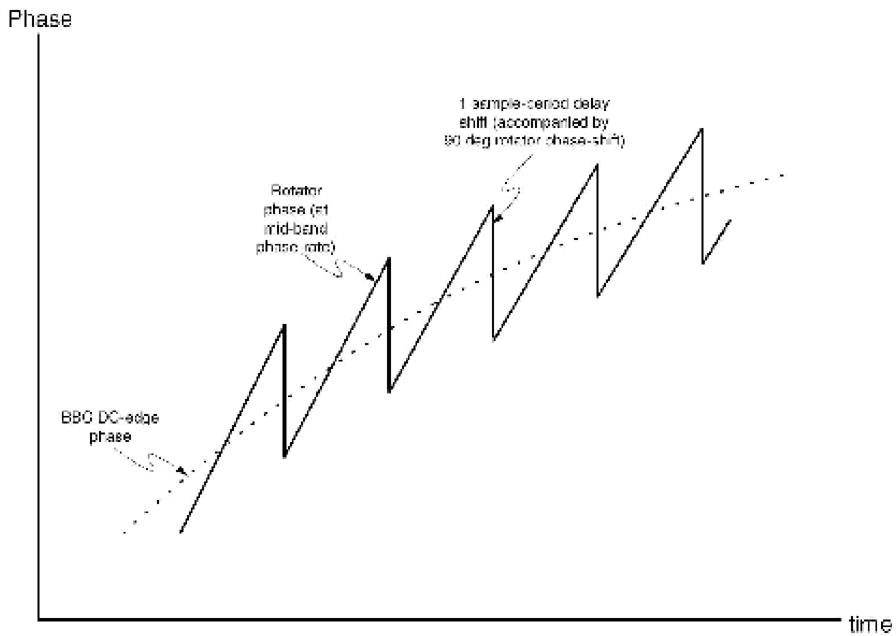
- Delay τ applied to one station
- Fringe rotation (rate $\dot{\tau}$) applied to the other
- cos/sin mixing of fringe rotation (Hilbert transforms: Bracewell Ch.12)
- output $=$ one complex number per lag
- **XF**



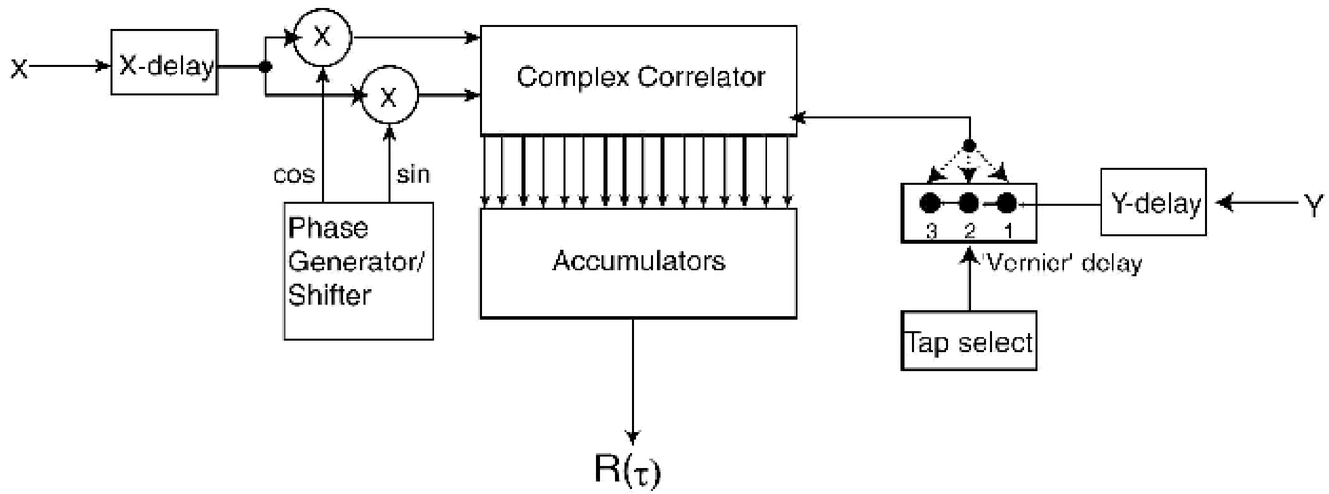
Running the fringe rotator

- track φ at band center (minimize phase offsets at edges)
- DC edge $\rightarrow$ band center: φ_{FR} increment $= \pm 2\pi(\frac{BW}{2})\delta_\tau \; = \; \pm\pi BW\delta_\tau$
- Step fringe rotation $\pm 90°$ at each instant delay shifted by $\pm\Delta_\tau$

$$\delta_\tau \rightarrow \delta_\tau \pm \Delta_\tau \; = \; \delta_\tau \pm \frac{1}{2BW}, \quad \varphi_{FR} \rightarrow \varphi_{FR} \pm \frac{\pi}{2}$$

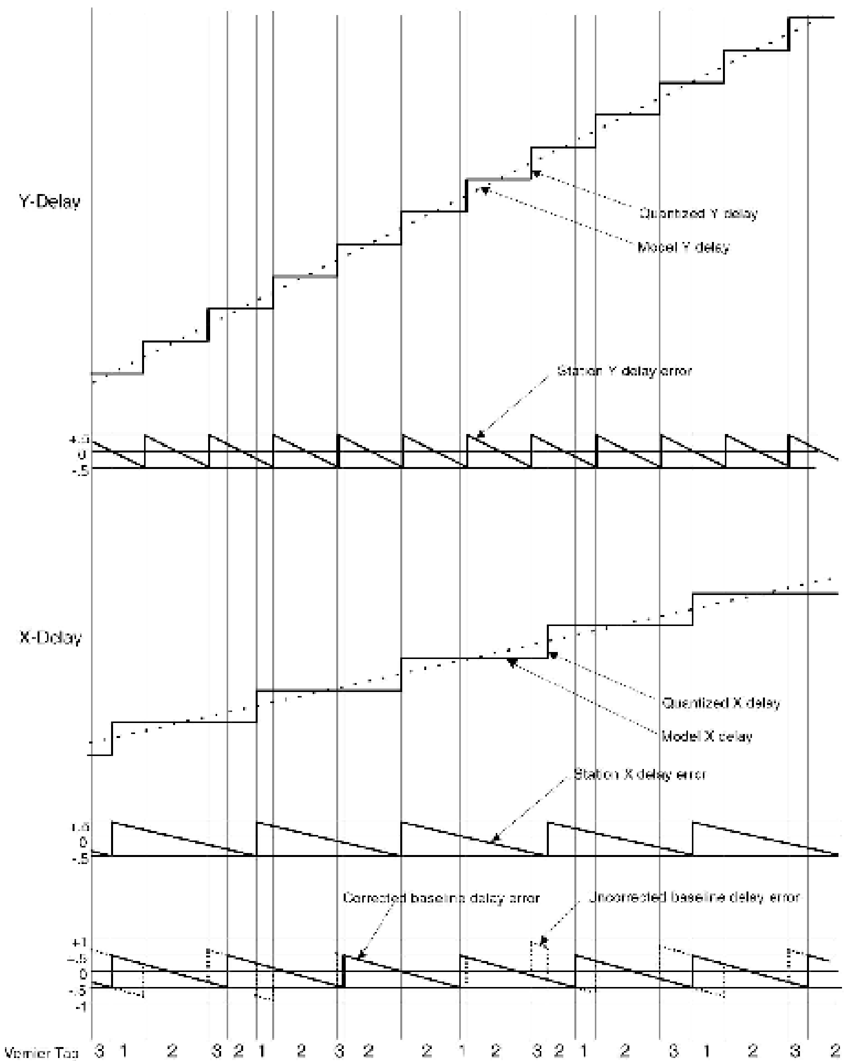


Station-based Correlator



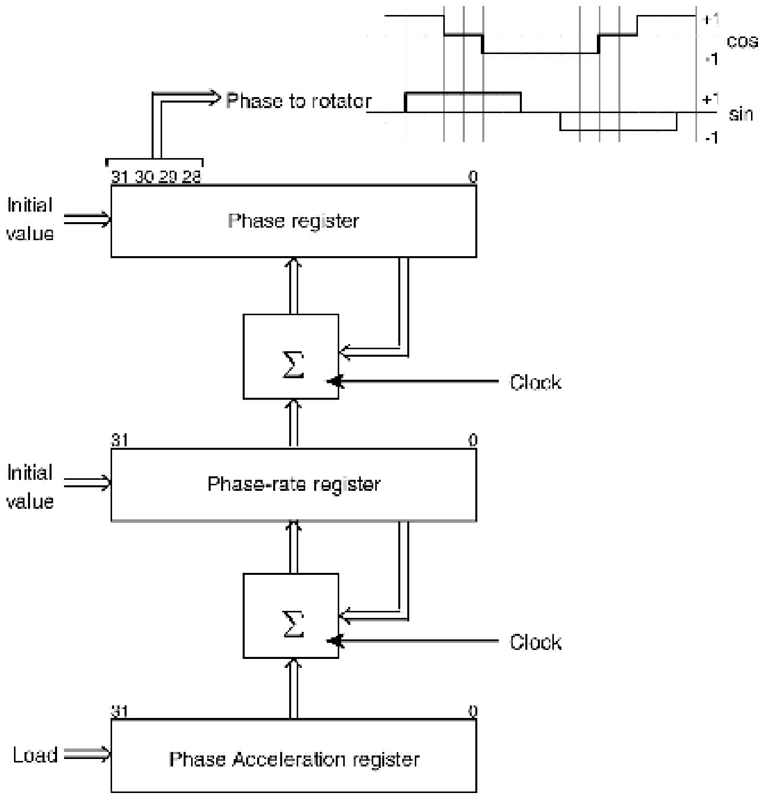
- Same as baseline XF, **PLUS** the "Vernier" delay tap

Station-based Delay Tracking



- $\exists$ periods of $|\delta\tau| > 0.5$ lag
  - Worse phase slopes accross band
- Vernier delay tap (± 1 lag) set to shift $|\delta\tau| > 0.5$ lag to $|\delta\tau| \leq 0.5$ lag

XF fringe-rotate both stations?

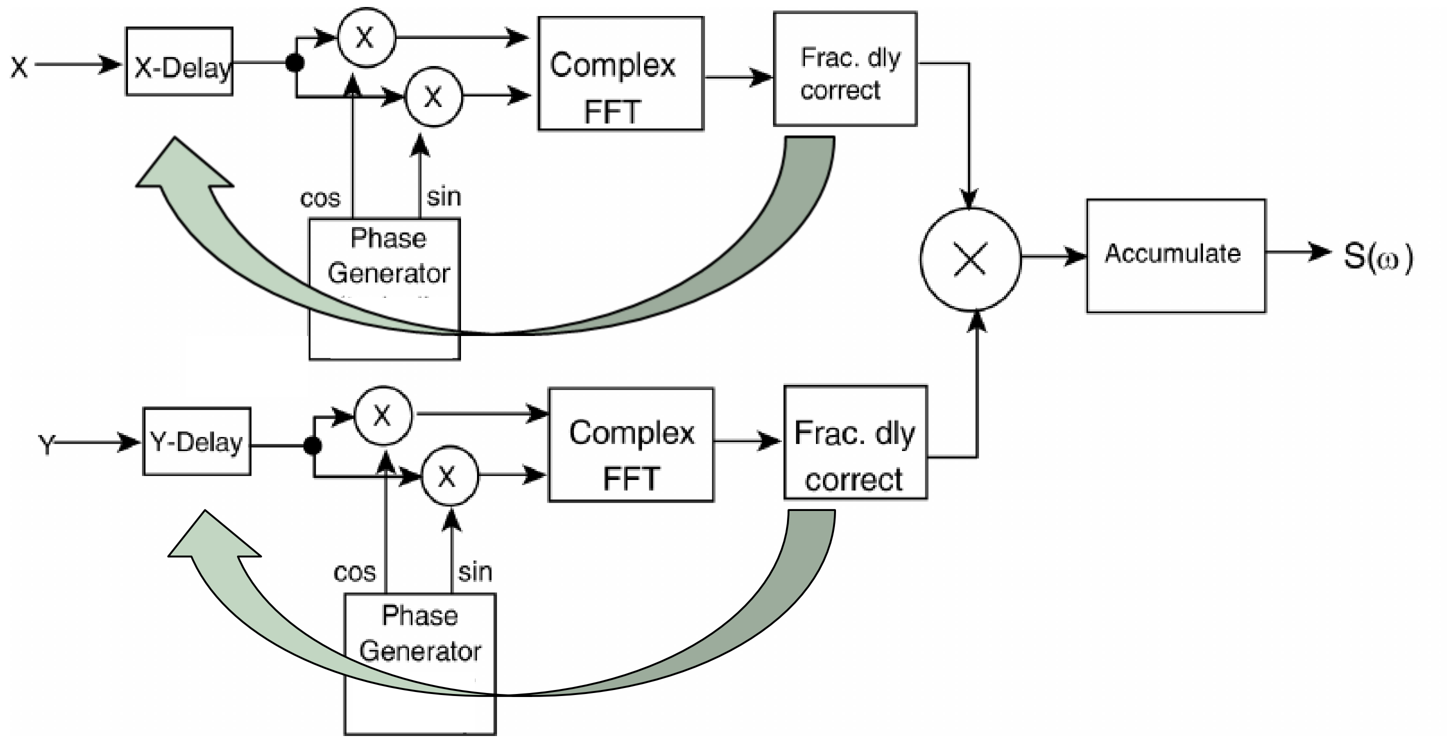


- 3-level sinusoids (+1, 0, −1)
  - doubled SNR loss
- Energy scattered into higher harmonics
- Fringe-rotating both stations would create two sets of spurious harmonics that correlate with each other
  - Delay-rate $\sim$ 0 events
- Updating phase "integration"
  - Speed-up
  - Oversampling

Correlations & Fourier Transforms

$$
\begin{tikzcd}
\mathbf{E}(t) \arrow[rr, Rightarrow] \arrow[dd, Rightarrow] \arrow[drr, bend left, "\mathbf{XF}"] \arrow[drr, bend right=60, "\mathbf{FX}"'] & & \mathbf{R}(\tau) = \mathbf{E}_1(t)\, \mathbf{E}_2^{\dagger}(t - \tau) \arrow[d, Rightarrow] \\
 & & \boldsymbol{P}(\omega) \\
\boldsymbol{E}(\omega) \arrow[r, Rightarrow] & \boldsymbol{E}_1(\omega)\, \boldsymbol{E}_2^{\dagger}(\omega) \arrow[ur, Rightarrow] &
\end{tikzcd}
$$

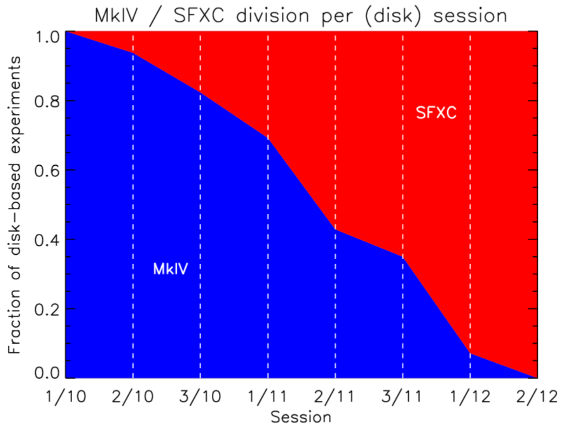
FX Station-based Correlator (SFXC)



- Fractional bit-shift delay via separate FFT earlier in delay compensation
- Quadrature rotation for removing *a priori* fringe rate
- A windowing step performed between fringe rate and station-based FFT
- The size of the 2 FFT’s independent of each other.

Correlator Evolution at JIVE (I)

- ENV MkIV Data Processor at JIVE
  - 16 stations (via tape, later Mark5 disk-pack playback)
  - Max. 1Gbps (but runs real-time regardless of input rate <1Gbps)
  - Max. 2048 frequency points per subband/pol
  - Byzantine rules for joint max. N{subbands, pols, frequency points}
  - Bespoke (irreplaceable) chips
- EVN Software Correlator at JIVE (SFXC)
  - Much more flexible, but not necessarily real-time
  - 1st user experiment correlated on SFXC from May/Jun 2010



Correlator Evolution at JIVE (II)

- Astronomy Gains enabled by SFXC
  - ~Arbitrary number of stations, total bit-rate, channel BW
    - Minimum t_{int} = 1 correlation FFT size in lag-space
  - Wide-field mapping (large $N_{freq.pt.}$ per SB/pol, short t_{int})
  - Multiple output phase centers per pointing
    - Delay/rate steering to N phase centers from within “internal” wide-field correlation
  - Pulsar gating/binning (1 gate, N equally spaced bins + out-of-gate bin)
    - Incoherent and coherent de-dispersion
  - Spectral zooming
  - Choice of spectral-windowing function (Hann, Hamming, top-hat, cosine)
  - Mixed input channel BWs → handling heterogeneous back-ends
  - External model files (e.g., near-field target or orbiting antenna)
  - Pure station-based fringe rotation to CoE; consistent cross-pols

Useful Links

- Correlators in general (slants towards geodetic perspective)
  - Whitney 2000, *IVS General Meeting 2000*

- EVN Correlators
  - MarkIV (XF)
    - Schilizzi et al. 2001, *Exper. Astron*., <u>12</u>, 49 (EVN MarkIV)
    - Whitney et al. 2004, *Rad. Sci.*, <u>39</u>, 1007 (Archit. & Algo.'s)
  - SFXC
    - Keimpema et al. 2015, *Exper. Astron*., <u>39</u>, 259
    - Keimpema et al. 2025, *Exper. Astron*., in press